Harmonics: Problem Solving

1. Determine the fundamental frequency (1st harmonic) of an open-ended air column that has a length of 67.5 cm.

$L = \frac{1}{2}\lambda_1$

$\lambda = 2L = 2(67.5)$

$\lambda_1 = 135\text{ cm}$

$v = \lambda f$

$340 = (1.35)f$

$f_1 = 252\text{ Hz}$

ANSWER: 252 Hz

2. Determine the length of an open-ended air column required to produce a fundamental frequency (1st harmonic) of 480 Hz.

$v = \lambda f_1$

$340 = \lambda_1(480)$

$\lambda_1 = 0.708$

$L = \frac{1}{2}\lambda_1 = \frac{1}{2}(.708)$

$L = 0.354\text{ m}$

ANSWER: 0.354m

3. Determine the first harmonic of an open air column whose fourth harmonic frequency is 1296 Hz.

1296 Hz

$v = \lambda_4 f_4$

$340 = \lambda_4(1296)$

$\lambda_4 = 0.262$

4th) $L = 2\lambda_4$

$L = 2(0.262) = 0.524$

1st) $L = \frac{1}{2}\lambda_1$

$0.524 = \frac{1}{2}\lambda_1$

$\lambda_1 = 1.048$

$v = \lambda f_1$

$f_1 = 324\text{ Hz}$

ANSWER: 324 Hz

4. An open-ended flute is being played at its second harmonic, 880Hz. The speed of sound through the flute is 350 m/s. Find the frequency of the first harmonic (the fundamental frequency) and the length of the flute.

$v = \lambda f$

$350 = \lambda_2(880)$

$\lambda_2 = 0.398\text{m}$

2nd) $L = \lambda_2$ $L = 0.398\text{m}$

1st) $L = \frac{1}{2}\lambda_1$

$0.398 = \frac{1}{2}\lambda_1$

$\lambda_1 = 0.795\text{m}$

$v = \lambda_1 f_1$

$f_1 = 440\text{Hz}$

ANSWER: 440 Hz, 0.398m.

5. Determine the fundamental frequency (1st harmonic) of an ~~open~~ closed-ended air column that has a length of 67.5 cm.

$L = \frac{1}{4}\lambda_1$

$67.5 = \frac{1}{4}\lambda_1$

$\lambda_1 = 270\,cm$

$v = \lambda_1 f_1$

$340 = 2.70\,(f_1)$

$f_1 = 126\,Hz$

ANSWER: 126 Hz

6. Determine the length of an ~~open~~ closed-ended air column required to produce a fundamental frequency (1st harmonic) of 480 Hz.

$v = \lambda_1 f_1$

$340 = \lambda_1\,(480)$

$\lambda_1 = 0.708\,m$

$L = \frac{1}{4}\lambda_1$

$L = 0.177\,m$

ANSWER: 0.177m

7. A closed-end organ pipe is used to produce a mixture of sounds. The third and fifth harmonics in the mixture have frequencies of 1100 Hz and 1833 Hz respectively. What is the frequency of the first harmonic played by the organ pipe?

$f_3 = 1100\,Hz$

$f_5 = 1833\,Hz$

3rd) $L = \frac{3}{4}\lambda_3$

$L = 0.232$

$v = \lambda_3 f_3$

$340 = \lambda_3\,(1100)$

$\lambda_3 = 0.30909$

$v = \lambda_5 f_5$

$340 = \lambda_5\,(1833)$

$\lambda_5 = 0.18549$

5th) $L = \frac{5}{4}\lambda_5$

$L = 0.232$

1st) $L = \frac{1}{4}\lambda_1$

$0.232 = \frac{1}{4}\lambda_1$

$\lambda_1 = 0.928\,m$

$v = \lambda f$

$f_1 = 366\,Hz$

ANSWER: 367 Hz

8. A closed-end pipe can make a first harmonic of 349.2 Hz. The speed of sound in the pipe is 350m/s. Find the length of the air column inside the pipe.

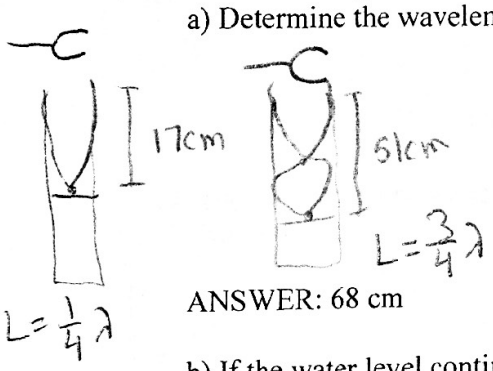
$f_1 = 349.2\ Hz$

$V = \lambda_1 f_1 \quad \lambda_1 = 1.002\ m$

$L = \frac{1}{4}\lambda_1 \quad \boxed{= 0.251\ m}$

ANSWER: 0.250 m

9. A tuning fork is held over the open end of a closed air column. The water level in the column is lowered. An increase in loudness is detected when the air column is 17cm long, and again when it is 51cm long.
a) Determine the wavelength of the sound produced by the tuning fork.



$17 = \frac{1}{4}\lambda$

$\lambda = 68cm$

$51 = \frac{3}{4}\lambda$

$\lambda = 68cm$

ANSWER: 68 cm

b) If the water level continues to be lowered, at what level will the next increase in loudness be heard?

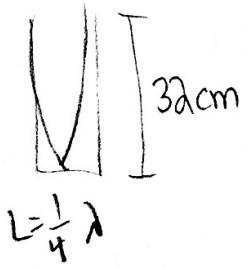
5th harmonic

$L = \frac{5}{4}\lambda = \frac{5}{4}(68)$

$L = 85cm$

ANSWER: 85 cm

10. The first resonance length of an air column is 32cm.
a) Determine the second and third resonance lengths, if the column is closed at one end.



$L = \frac{1}{4}\lambda$

$32 = \frac{1}{4}\lambda$

$\lambda = 128\ cm$

2nd resonance length (3rd harmonic)

$L = \frac{3}{4}\lambda$

3rd resonance length (5th harmonic)

$L = \frac{5}{4}\lambda$

$L_2 = 96cm$

$L_3 = 160cm$

ANSWER: 96 cm, 160 cm

b) Determine the second and third resonance lengths, if the column is open at both ends.

$L = \frac{1}{2}\lambda$

$32 = \frac{1}{2}\lambda$

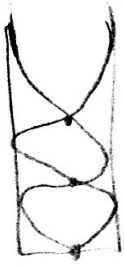
$\lambda = 64\text{cm}$

2nd resonance $L = \lambda$ $\quad L_2 = 64\text{cm}$

3rd resonance $L = \frac{3}{2}\lambda$ $\quad L_3 = 96\text{cm}$

ANSWER: 64 cm, 96 cm

11. The third resonance length of a closed air column resonating to a tuning fork is 95cm. Determine the first and second resonant lengths.



$L = 95\text{cm}$

$L = \frac{5}{4}\lambda$

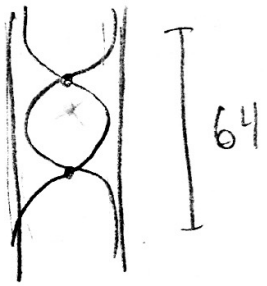
$\lambda = 118.75$

1st $L = \frac{1}{4}\lambda$ $\quad L_1 = 29.69\text{ cm}$

2nd $L = \frac{3}{4}\lambda$ $\quad L_2 = 89.06\text{cm}$

ANSWER: 29.69 cm, 89.06 cm

12. The second resonance length of an air column, open at both ends and resonating to a fixed frequency, is 64 cm. Determine the first and third resonance lengths.



$L = \lambda \quad \lambda = 64\text{cm}$

1st) $L = \frac{1}{2}\lambda = 32\text{cm}$

3rd) $L = \frac{3}{2}\lambda = 96\text{cm}$

ANSWER: 32 cm, 96 cm

13. The frequency of a tuning fork is unknown. A student uses an open ended air column at room temperature and finds resonances spaced by 20.2 cm. What is the frequency of the tuning fork? Speed of sound is 347 m/s.

$L_2 - L_1 = 20.2$

$\lambda - \frac{1}{2}\lambda = 20.2$

$\frac{1}{2}\lambda = 20.2$

$\lambda = 40.2\text{cm}$

$v = \lambda f$

$347 = (0.402) f$

$f = 863\text{ Hz}$

ANSWER: 859 Hz

14. An organ pipe, open at both ends, needs to resonate in its fundamental mode with a frequency of 128 Hz. The organ has been designed to be played at 22 degrees Celsius (speed of sound = 344 m/s)

a) How long does the organ pipe need to be?

$v = \lambda f$

$344 = \lambda(128)$

$\lambda = 2.6875$

$L = \frac{1}{2}\lambda$ → $L = 1.34\text{m}$

ANSWER: 1.34 m

b) If this pipe is closed at one end by a stopper, at what fundamental frequency will it resonate?

$L = 1.34\text{m}$

$L = \frac{1}{4}\lambda$

$\lambda = 5.375$

$v = \lambda f$

$344 = 5.375(f)$

$f = 64\text{Hz}$

ANSWER: 64 Hz

15. The 440 Hz tuning fork is used with a resonating column to determine the velocity of sound in helium gas. If the spacing between resonances is 110 cm, what is the velocity of sound in helium?

$L_2 - L_1 = 110\text{cm}$

$\frac{3}{4}\lambda - \frac{1}{4}\lambda = 110$

$\frac{1}{2}\lambda = 110$

$\lambda = 220$

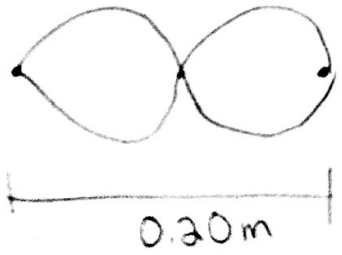
$v = \lambda f = (2.20)(440)$

$v = 968\text{m/s}$

ANSWER: 970 m/s

16. A standing wave in a guitar string has a frequency of 28 Hz at the second harmonic.

a) If the wavelength is 0.20m, what is the distance between nodes?



btw nodes = 0.10m

ANSWER: 0.10m

b) What is the speed of the waves that make up the standing wave?

$v = \lambda f$
$= (0.20)(28)$
$v = 5.6\,m/s$

ANSWER: 5.6 m/s

c) What would be the frequency of a rope vibrating at the third harmonic?

3rd $L = \frac{3}{2}\lambda$

$0.20 = \frac{3}{2}\lambda$

$\frac{.4}{3} = \lambda$

$v = \lambda f$
$5.6 = (.133) f$
$f = 42\,Hz$

ANSWER: 42 Hz

17. An 85 cm long guitar string is plucked and vibrates at the fourth harmonic. What is the frequency of the sound produced if the speed ~~of sound~~ vibration is 332 m/s?

4th) $L = 2\lambda$

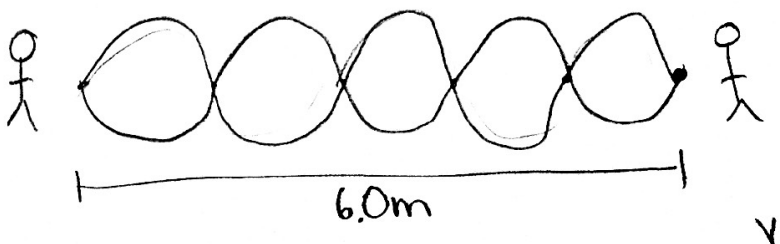
$85cm = 2\lambda$

$\lambda = 42.5\,cm$

$v = \lambda f$
$332 = (.425) f$
$f = 780\,Hz$

ANSWER: 780 Hz

18. Two children playing with a 6.0 m long skipping rope produce a standing wave pattern with 5 "loops". If the skipping rope is vibrating at 85 vibrations per minute, what is the speed of the vibration producing the standing wave pattern?



$L = 2.5\lambda$
$\lambda = 2.4$

$f = \frac{85}{min} \cdot \frac{1\,min}{60s} = 1.42\,Hz$

$v = \lambda f$
$= (2.4)(1.42)$
$v = 3.4\,m/s$

ANSWER: 3.4 m/s

19. A violin string that is 50.0 cm long has a fundamental frequency of 440 Hz. What is the speed of the waves on this string?

$L = 50$ $\quad f_1 = 440\,Hz$

$L = \frac{1}{2}\lambda$

$\lambda = 100\,cm$

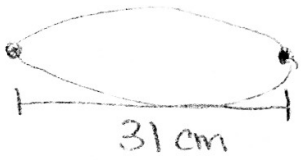
$v = \lambda f$

$v = (1)(440)$

$v = 440\,m/s$

ANSWER: 440 m/s

20. What are the first three harmonics of a note produced on a 31.0 cm long violin string if waves on this string have a speed of 274.4 m/s?



$v = 274.4\,m/s$

(1st) $L = \frac{1}{2}\lambda$

$\lambda = 62\,cm$

$v = \lambda f$

$274.4 = (.62) f$

$f_1 = 442.6\,Hz$

ANSWER: 442.6 Hz, 885.2 Hz, 1327.8 Hz

(2nd) $L = \lambda$

$31 = \lambda$

$274.4 = (.31) f$

$f_2 = 885.2\,Hz$

(3rd) $L = \frac{3}{2}\lambda$

$31 = \frac{3}{2}\lambda$

$274.4 = (.207) f$

$f_3 = 1326\,Hz$